RIVAS Training Workshop – “Reducing railway induced ground vibration by interventions on the transmission path”
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Mitigation of vibrations by stiff wave barriers: physical mechanisms

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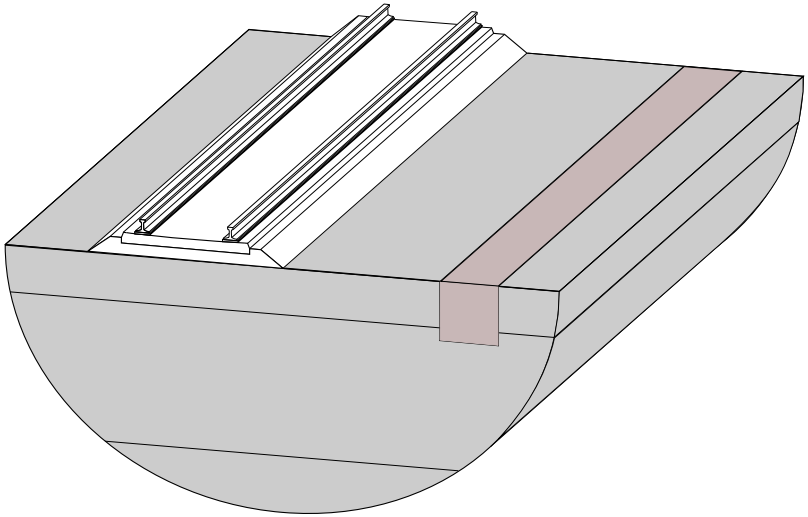
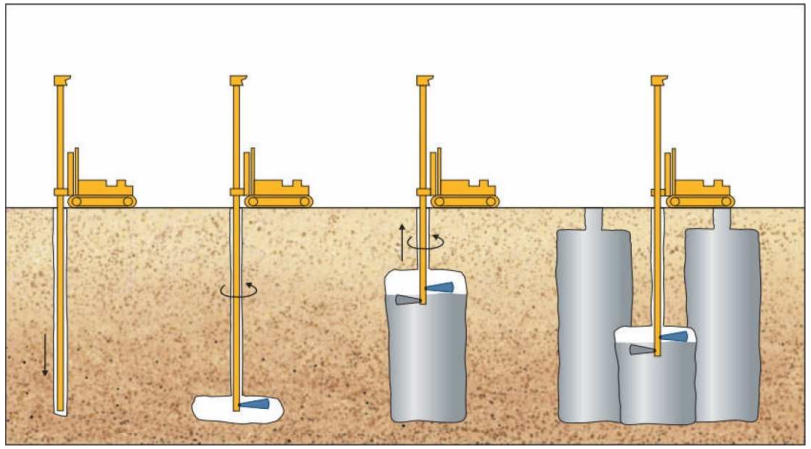
Introduction

Stiff wave barrier next to the track

- Construction of a jet grouting wall next to the track
- Wave impeding barrier for railway induced vibrations





- Common construction techniques:
  - deep vibro compaction
  - gravel/cement columns
  - hydraulic fracture injection with stable cement–bentonite mixtures
  - . . .

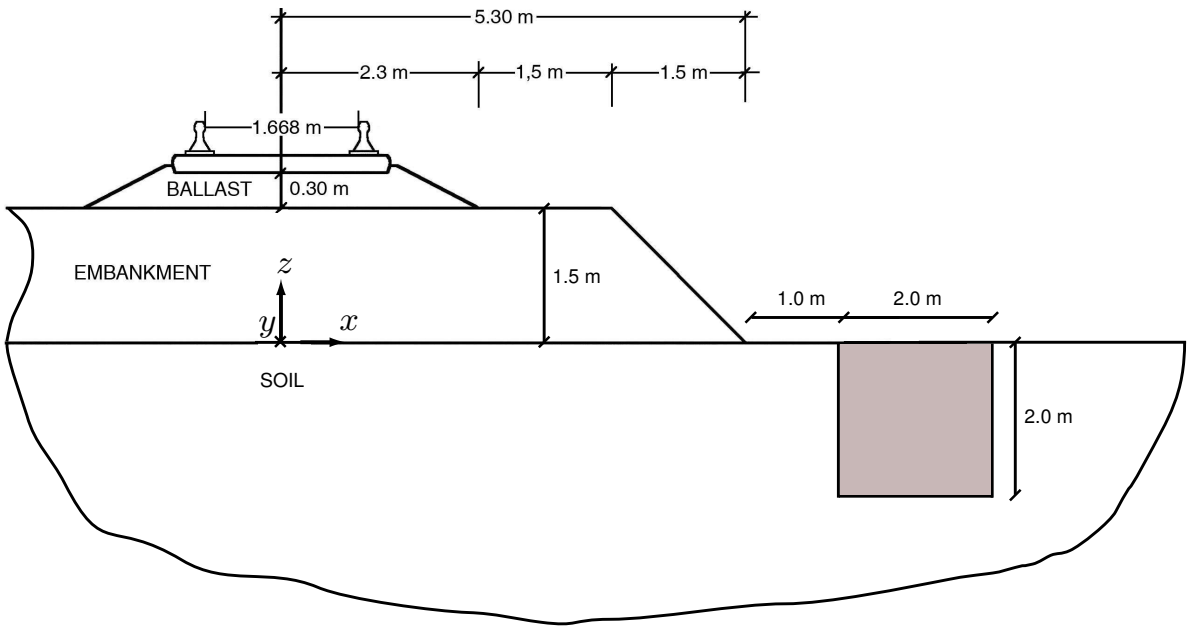
Introduction

Stiff wave barrier next to the track

- Block of stiffened soil in a homogeneous halfspace (e.g. by means of jet grouting)

	C_s [m/s]	C_p [m/s]	β_s [−]	β_p [−]	ρ [kg/m^3]
Halfspace	200	400	0.025	0.025	2000
Stiffened soil	550	950	0.05	0.05	2000

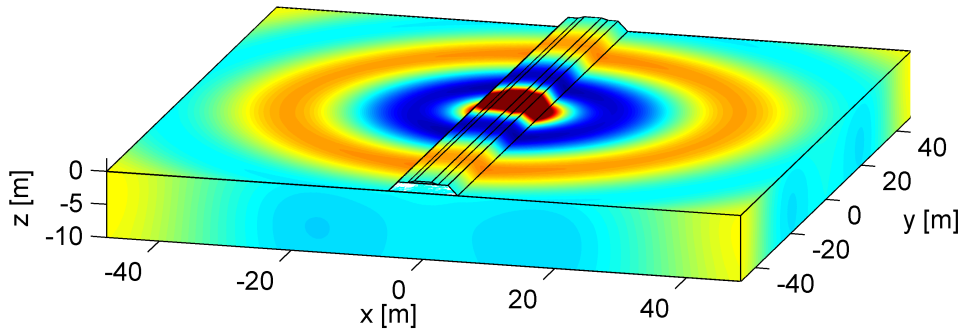
- Square cross–section: $w = h = 2\,\mathrm{m}$



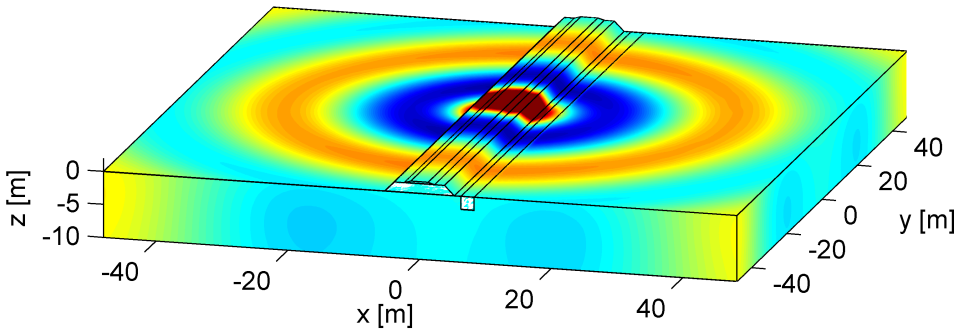
Transfer functions

- Real part of the vertical displacement field $\hat{u}_z(\mathbf{x}, \omega)$ at $5\,\mathrm{Hz}$

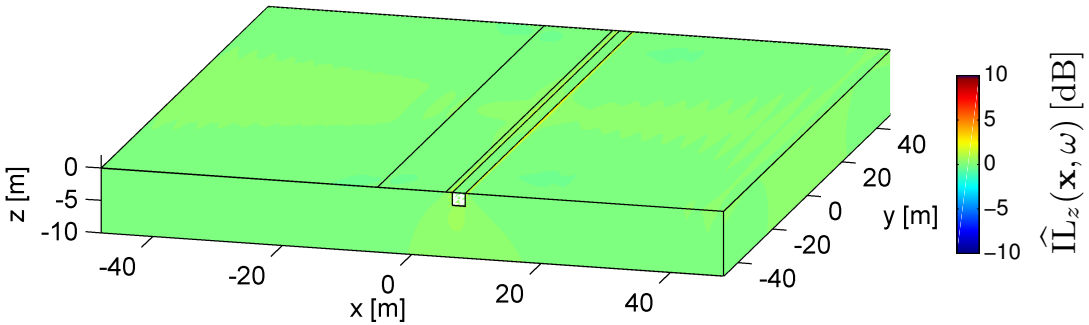
without barrier



with barrier



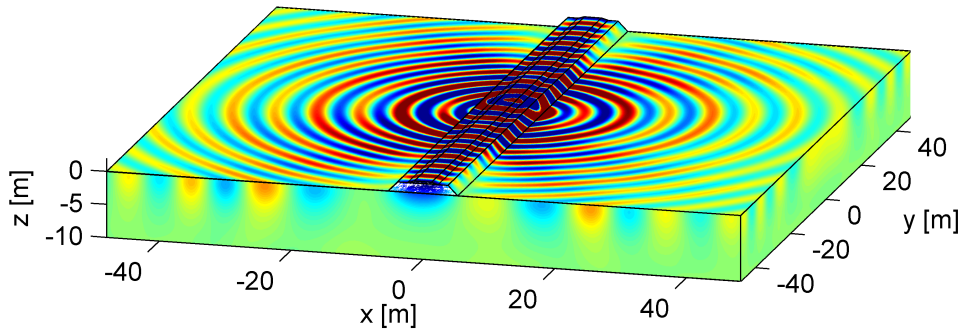
- Corresponding insertion loss $\widehat{\mathrm{IL}}_z(\mathbf{x}, \omega) = 20 \log_{10} \frac{|\hat{u}_z^{\mathrm{ref}}(\mathbf{x},\omega)|}{|\hat{u}_z(\mathbf{x},\omega)|}$



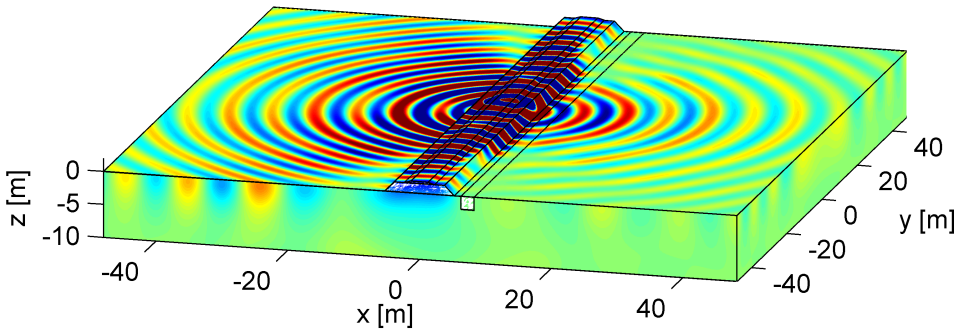
Transfer functions

- Real part of the vertical displacement field $\hat{u}_z(\mathbf{x}, \omega)$ at $30\,\mathrm{Hz}$

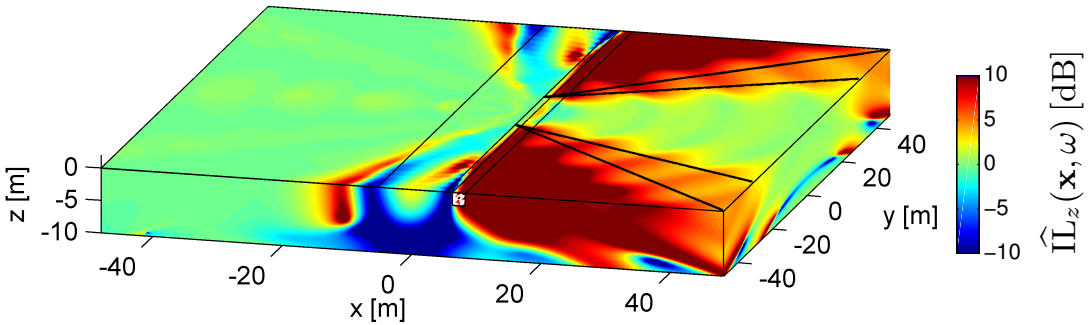
without barrier



with barrier



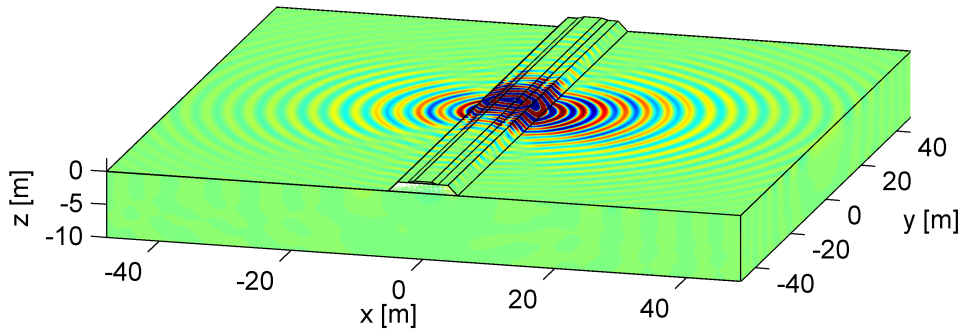
- Corresponding insertion loss $\widehat{\mathrm{IL}}_z(\mathbf{x}, \omega) = 20 \log_{10} \frac{|\hat{u}_z^{\mathrm{ref}}(\mathbf{x},\omega)|}{|\hat{u}_z(\mathbf{x},\omega)|}$



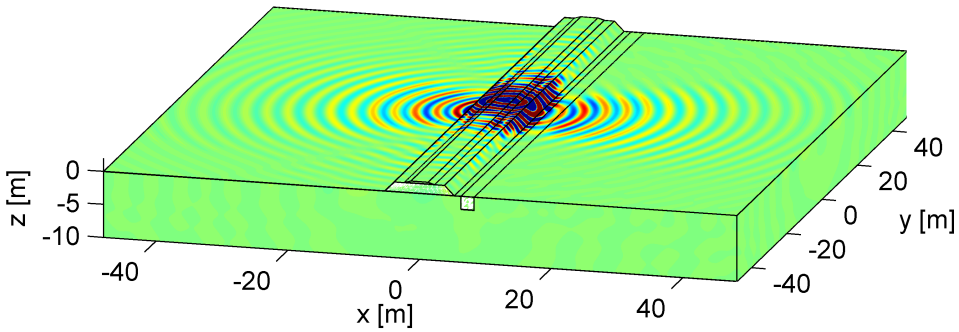
Transfer functions

- Real part of the vertical displacement field $\hat{u}_z(\mathbf{x}, \omega)$ at $60\,\mathrm{Hz}$

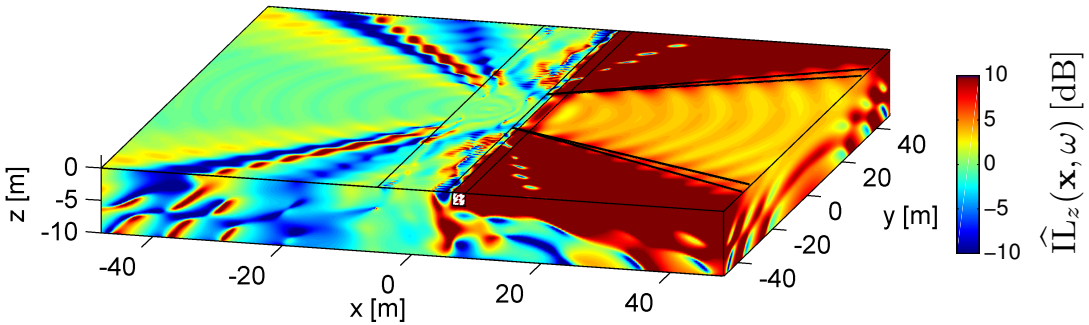
without barrier



with barrier



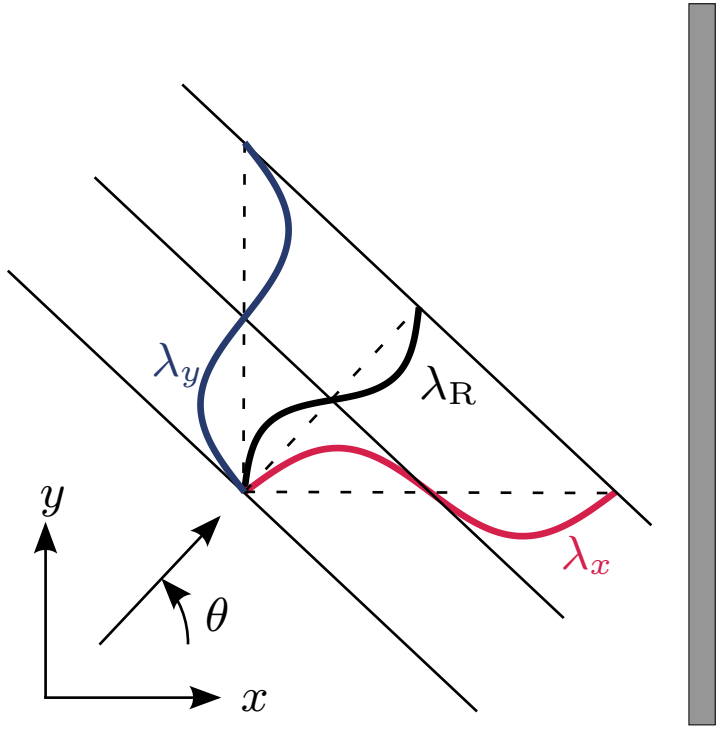
- Corresponding insertion loss $\widehat{\mathrm{IL}}_z(\mathbf{x}, \omega) = 20 \log_{10} \frac{|\hat{u}_z^{\mathrm{ref}}(\mathbf{x},\omega)|}{|\hat{u}_z(\mathbf{x},\omega)|}$



Transfer functions

Plane wave propagation

- Cylindrical wavefield can be decomposed into plane waves, satisfying the dispersion relation $\frac{1}{\lambda_x^2} + \frac{1}{\lambda_y^2} = \frac{1}{\lambda_{\mathrm{R}}^2}$, where $\lambda_{\mathrm{R}} = 2\pi \frac{C_{\mathrm{R}}}{\omega}$ is the Rayleigh wavelength.

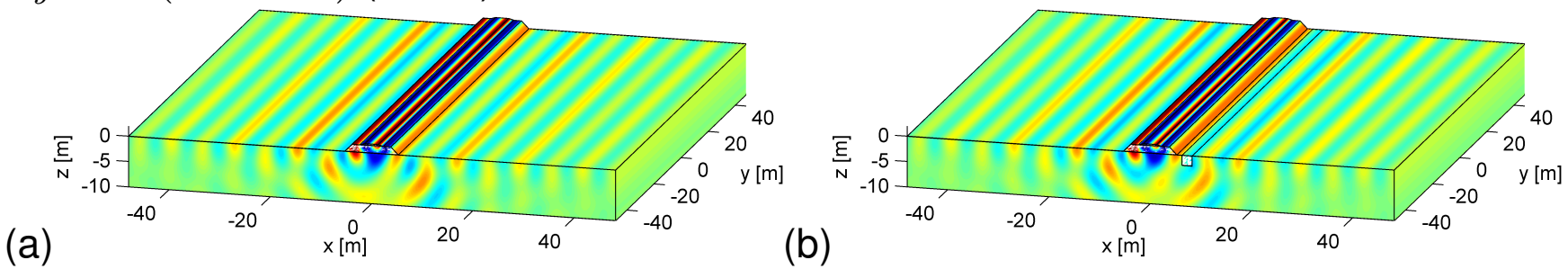


- Propagating plane waves are characterized by $\lambda_{\mathrm{R}} \leqslant \lambda_y \leqslant \infty$:
  - $\theta = 0 \Rightarrow \lambda_x = \lambda_{\mathrm{R}}, \lambda_y = \infty$
  - $\theta = \pi/2 \Rightarrow \lambda_y = \lambda_{\mathrm{R}}, \lambda_x = \infty$

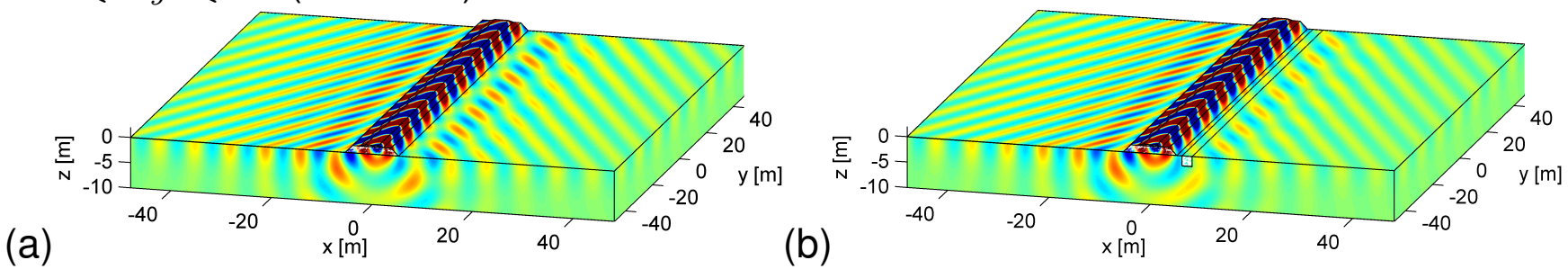
Transfer functions

Plane wave propagation

- Rayleigh wave propagation: $\frac{1}{\lambda_x^2} + \frac{1}{\lambda_y^2} = \frac{1}{\lambda_{\mathrm{R}}^2}$
- (a) Without and (b) with stiff wave barrier:
  - $\lambda_y = \infty (\lambda_x = \lambda_{\mathrm{R}})$ $(\theta = 0)$



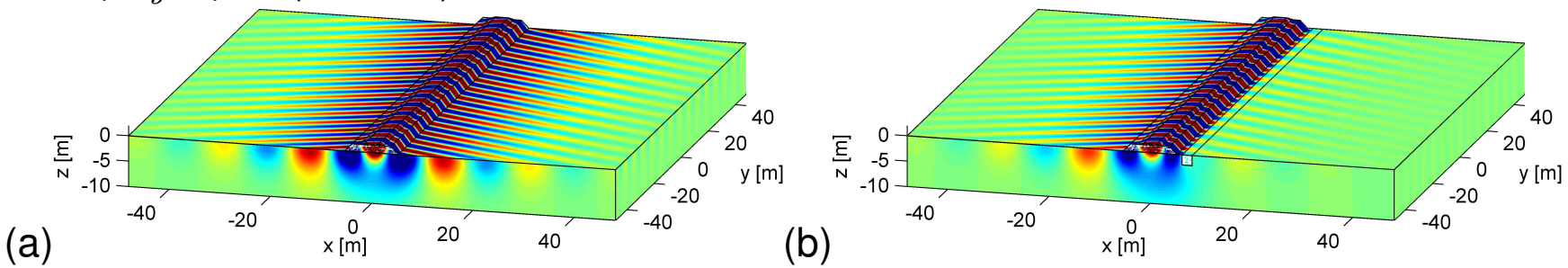
  - $\lambda_{\mathrm{R}} \leqslant \lambda_y \leqslant \infty$ $(\theta = 0.50)$



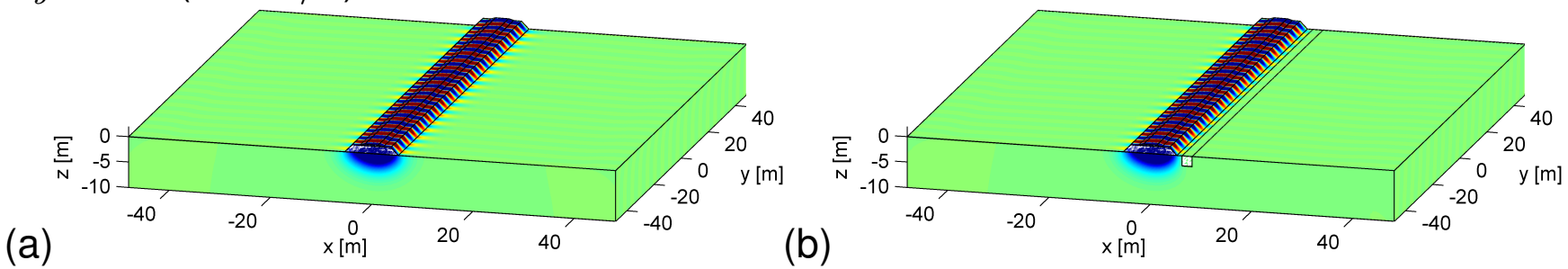
Transfer functions

Plane wave propagation

- Rayleigh wave propagation: $\frac{1}{\lambda_x^2} + \frac{1}{\lambda_y^2} = \frac{1}{\lambda_{\mathrm{R}}^2}$
- (a) Without and (b) with stiff wave barrier:
  - $\lambda_{\mathrm{R}} \leqslant \lambda_y \leqslant \infty$ $(\theta = 1.2)$



  - $\lambda_y < \lambda_{\mathrm{R}}$ $(\theta = \pi/2)$



Transfer functions

Interaction of Rayleigh waves in the soil and bending waves in the stiff wave barrier

- Rayleigh wave dispersion curve (black line):

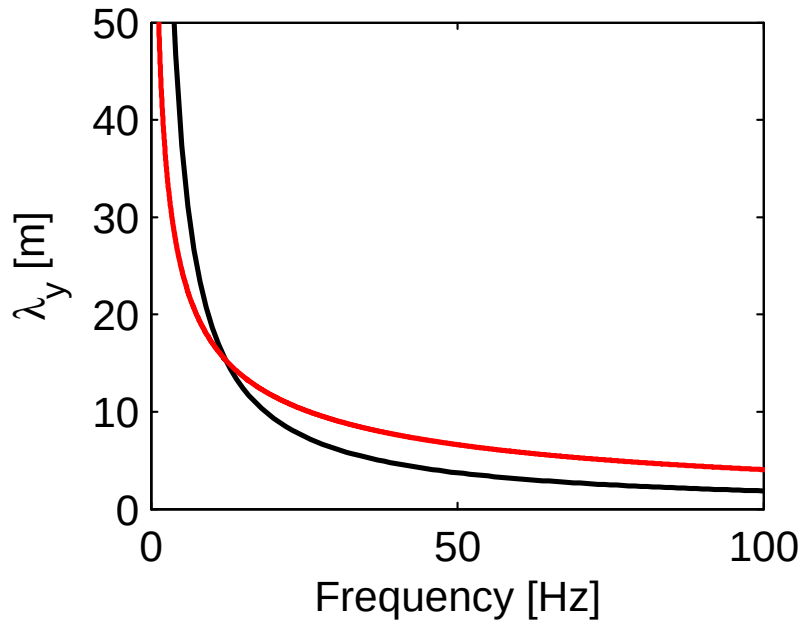
$$\lambda_{\mathrm{R}} = 2\pi \frac{C_{\mathrm{R}}}{\omega}$$

- Euler-Bernoulli beam theory in (λ_y, ω)-domain:

$$\left(-\rho A \omega^2 + EI \left(\frac{2\pi}{\lambda_y}\right)^4\right) \tilde{u}_z(\lambda_y, \omega) = \tilde{f}(\lambda_y, \omega)$$

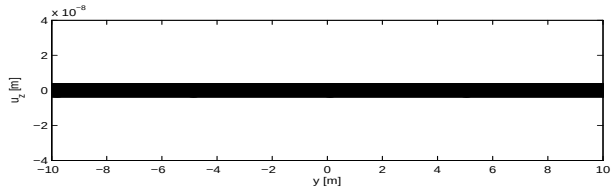
- Free bending wave dispersion curve (red line):

$$\lambda_{\mathrm{b}} = \frac{2\pi}{\sqrt{\omega}} \left(\frac{EI}{\rho A}\right)^{1/4}$$

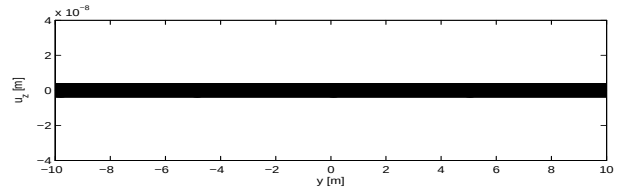


$\Rightarrow \tilde{u}_z(\lambda_y, \omega) \propto 0$ for $\lambda_y < \lambda_{\mathrm{b}}$

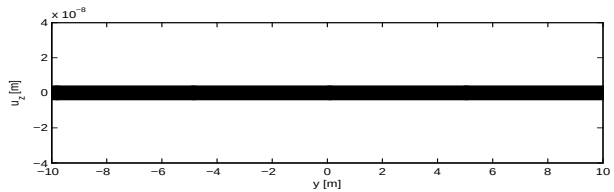
$\lambda_y = \infty$



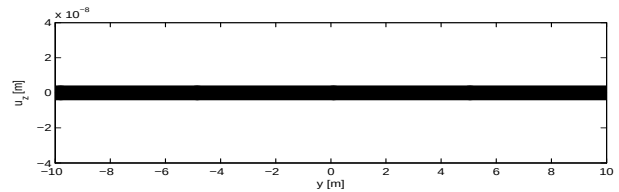
$\lambda_y = \lambda_{\mathrm{b}}$



$\lambda_y = 2/3\lambda_{\mathrm{b}}$



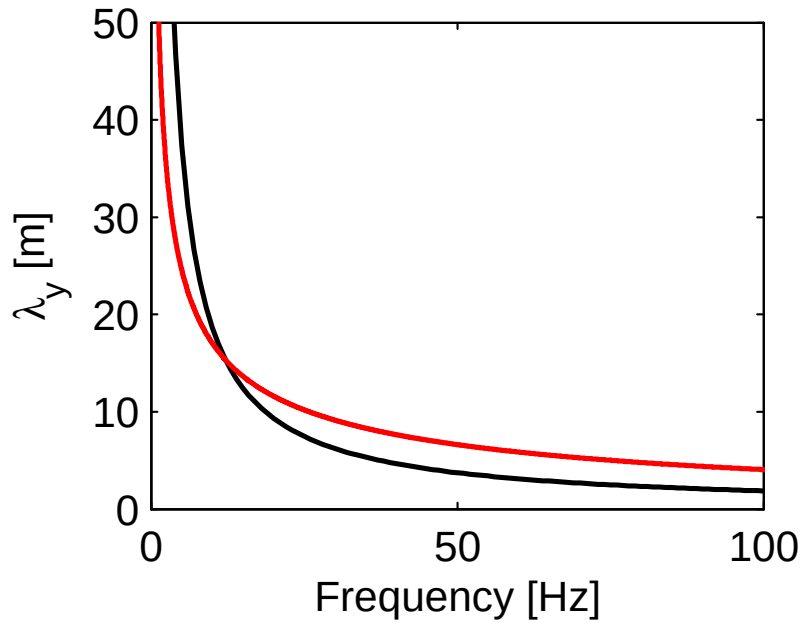
$\lambda_y = 1/3\lambda_{\mathrm{b}}$



Transfer functions

Interaction of Rayleigh waves in the soil and bending waves in the stiff wave barrier

- $f < f_\mathrm{c}$: $\lambda_\mathrm{b} < \lambda_\mathrm{R} \Rightarrow$ Rayleigh wave propagates unhindered through the block of stiffened soil
- $f > f_\mathrm{c}$: $\lambda_\mathrm{b} > \lambda_\mathrm{R} \Rightarrow$ wavefield is partially transmitted, partially blocked
  - $\lambda_y > \lambda_\mathrm{b}$: plane waves are transmitted ($\widehat{\mathrm{IL}}_z(\mathbf{x}, \omega) \sim 0\,\mathrm{dB}$)
  - $\lambda_y < \lambda_\mathrm{b}$: transmission of plane waves is impeded by the block of stiffened soil



- Critical frequency f_c (intersection of the Rayleigh wave and the free bending wave dispersion curves):

$$f_\mathrm{c} = \frac{\omega_\mathrm{c}}{2\pi} = \frac{C_\mathrm{R}^2}{2\pi}\sqrt{\frac{\rho A}{EI}} = \frac{C_\mathrm{R}^2}{2\pi h}\sqrt{\frac{12\rho}{E}} = 12\,\mathrm{Hz}$$

Transfer functions

Interaction of Rayleigh waves in the soil and bending waves in the stiff wave barrier

- The propagating plane waves $\lambda_y > \lambda_\mathrm{R}$ are characterized by a wave propagation direction $\theta = \sin^{-1}(\lambda_\mathrm{R}/\lambda_y)$. A reduction of vibration levels in the spatial domain will only be obtained in an area delimited by a critical angle $\theta_\mathrm{c}(\omega) = \sin^{-1}(\lambda_\mathrm{R}/\lambda_\mathrm{b})$:

$$\sin\theta_\mathrm{c} = \frac{C_\mathrm{R}}{\sqrt{\omega}}\left(\frac{\rho A}{EI}\right)^{1/4} = \frac{C_\mathrm{R}}{\sqrt{\omega h}}\left(\frac{12\rho}{E}\right)^{1/4}$$